Treelet Covariance Smoothers

Estimation of Genetic Parameters

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Overview

- Principle Component Analysis

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- Motivation in Statistical Genetics

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- Existing Methods

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- New Methods & Results

Principle Component Analysis (PCA)

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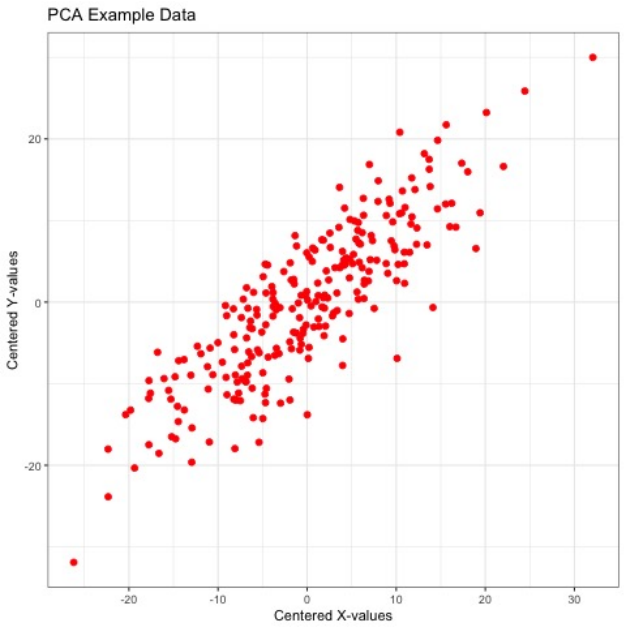

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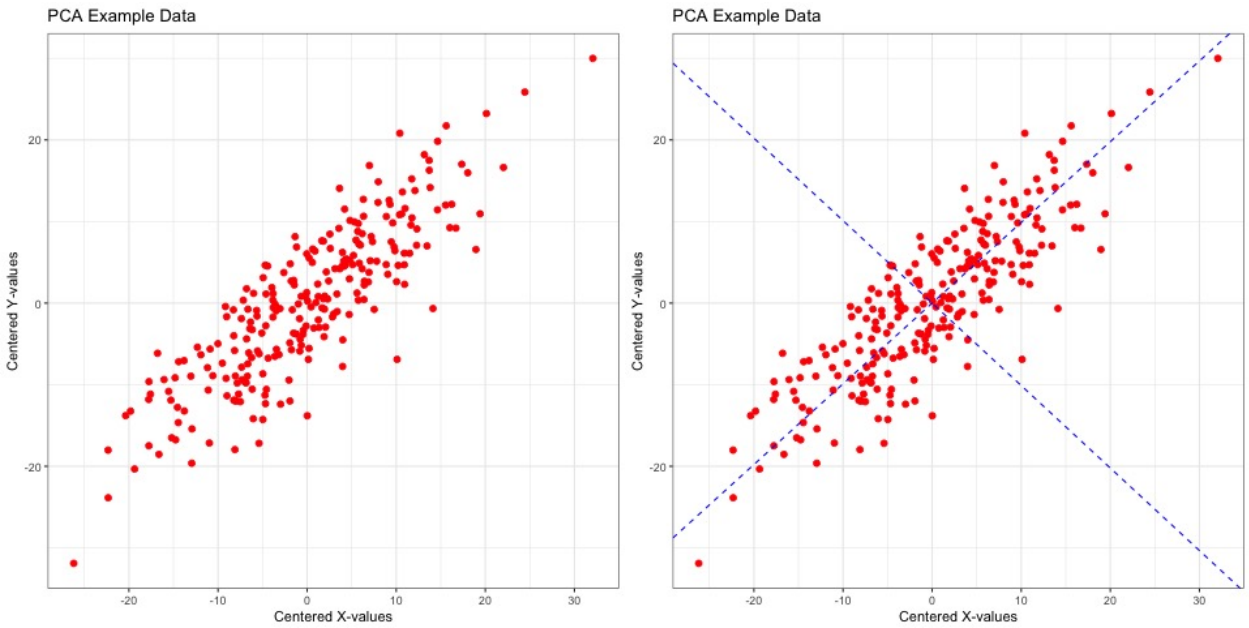

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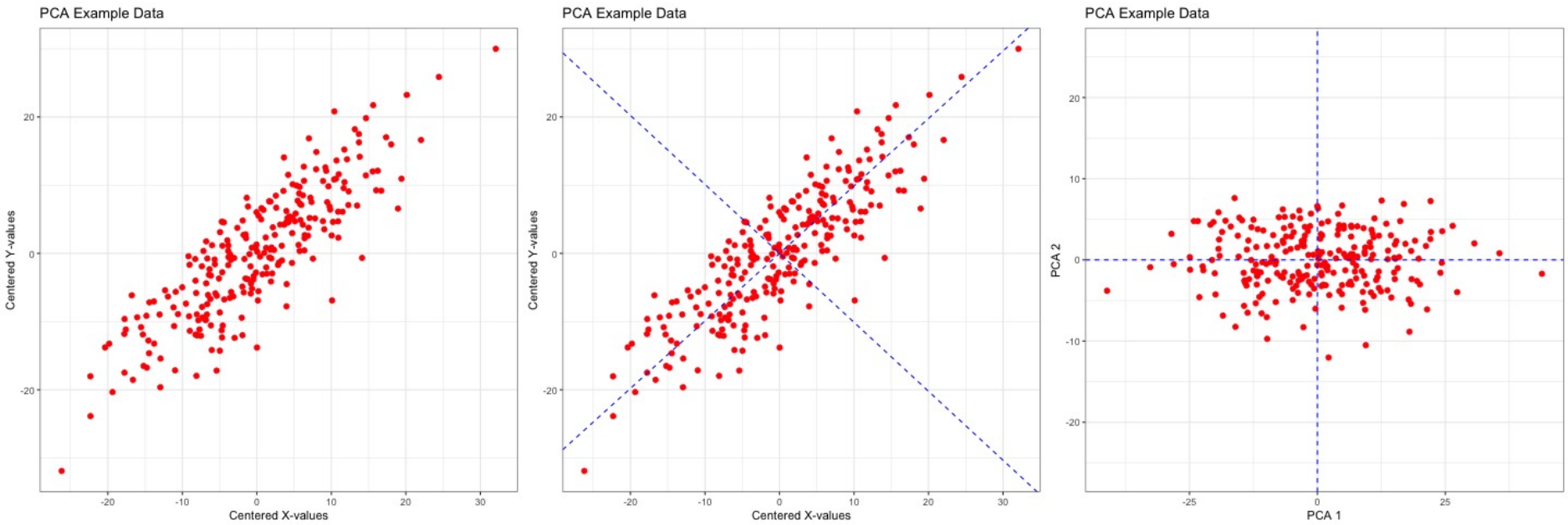

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- Idea: use PCA to set the data on top of a more "natural" basis

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- Once all individuals are processed, enforce sparsity to improve the estimator

TCS Properties

P1

P2

P3

P4

P5

P6

P7

P8

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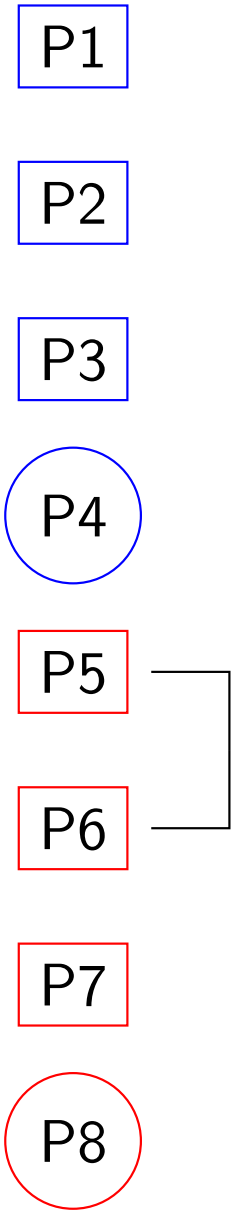

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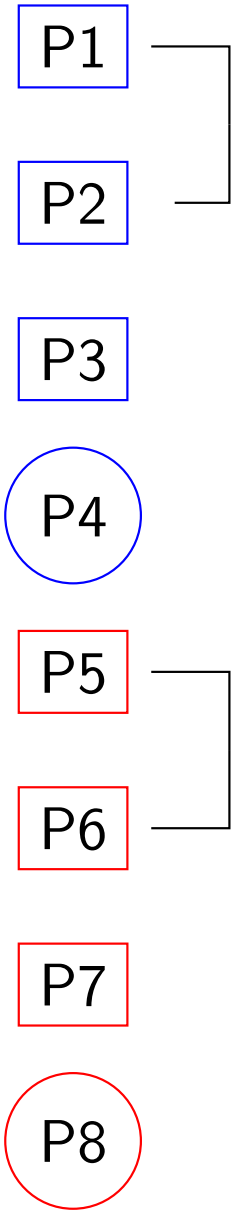

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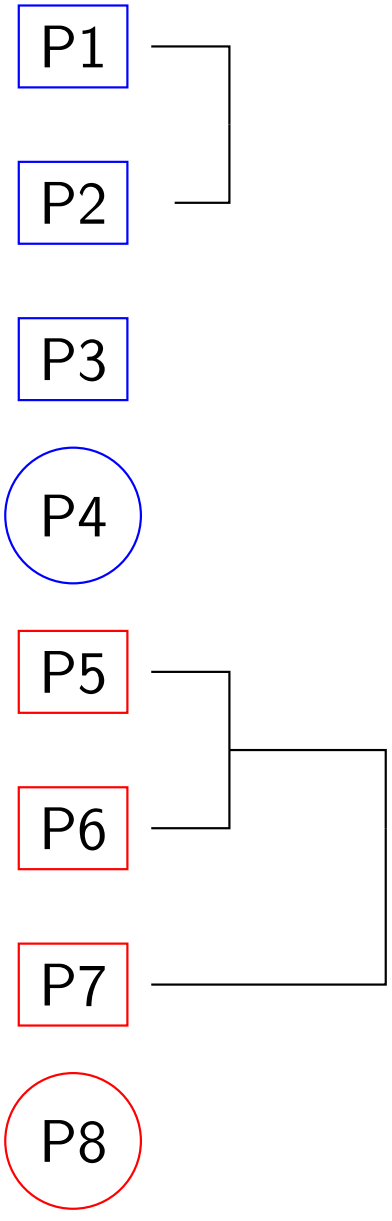

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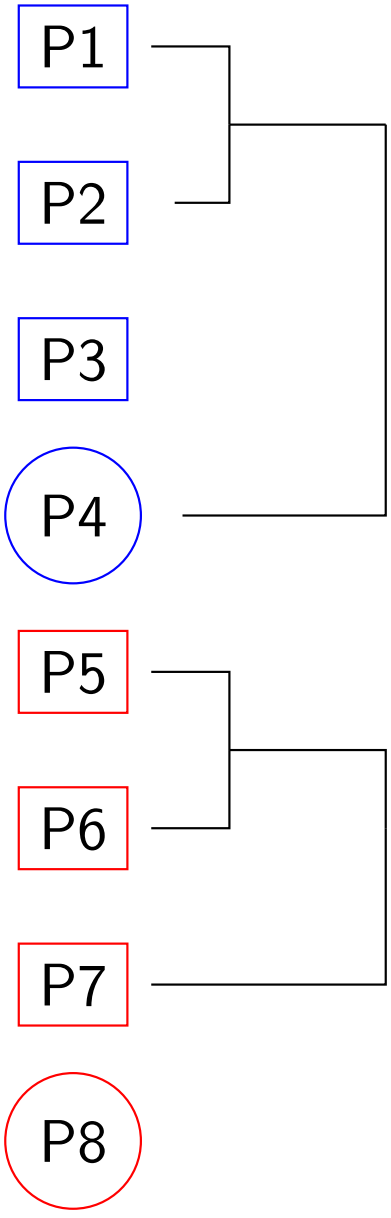

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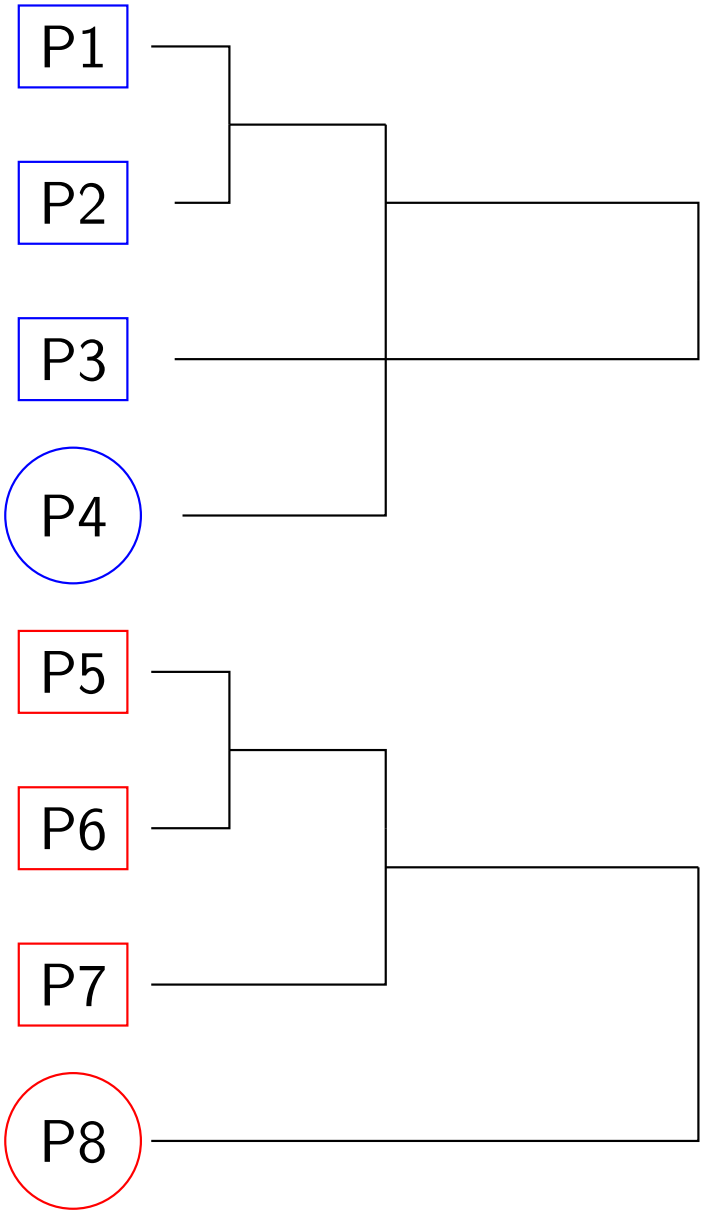

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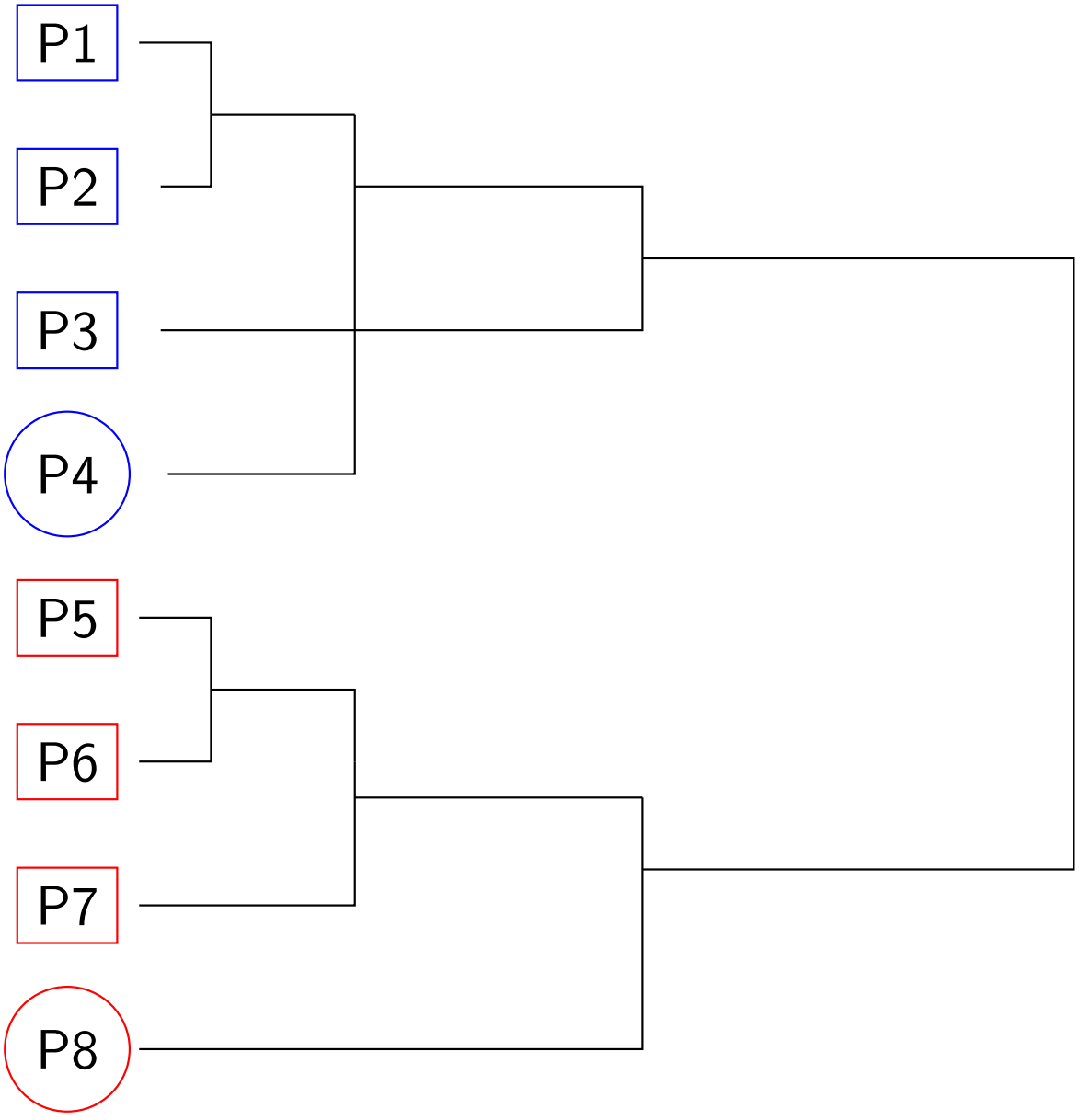

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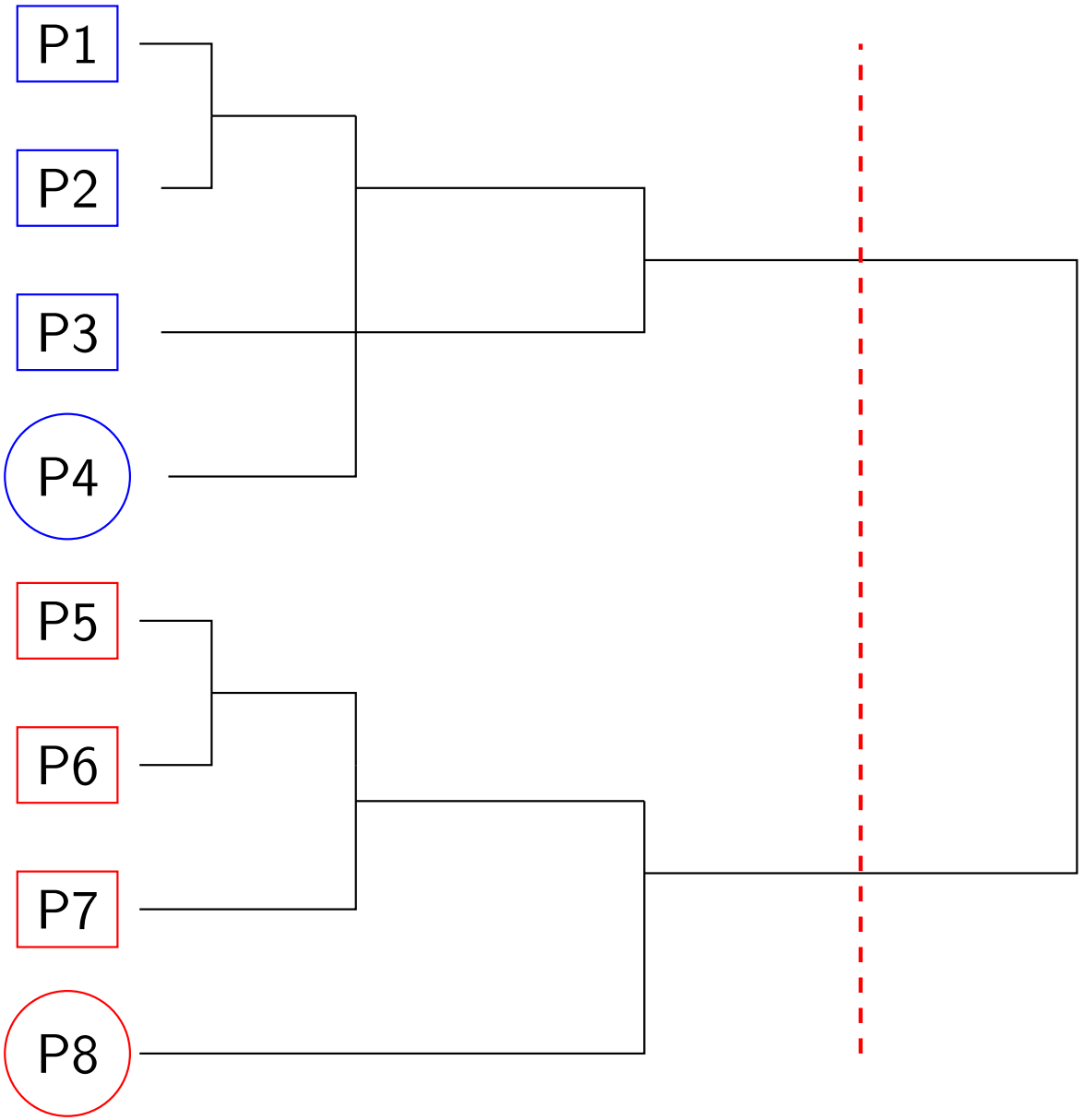

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- Further enforce sparsity by thresholding estimates
  - Treelet Covariance Blocked Smoothing (TCBS)

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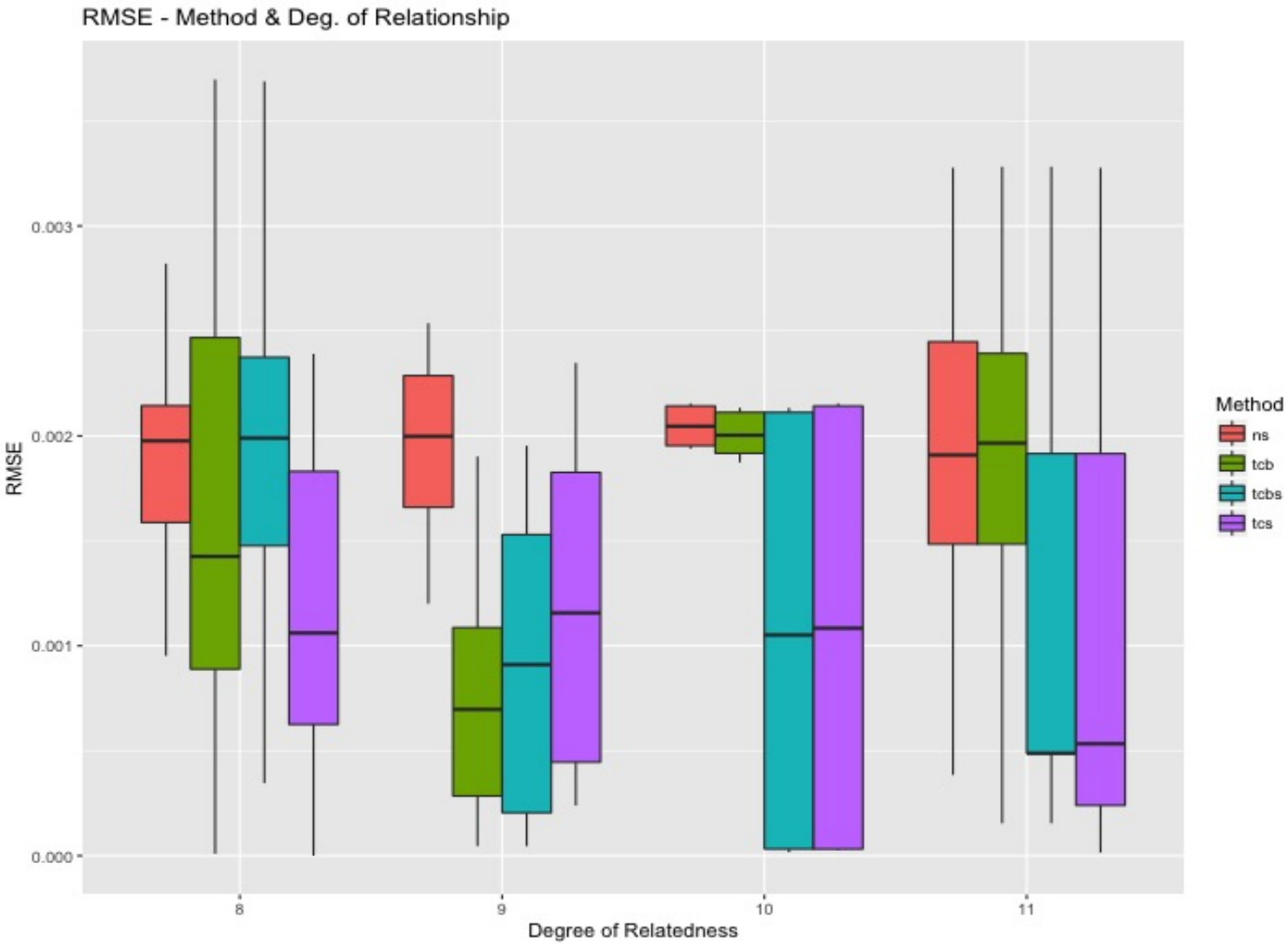
- We can enforce sparsity by further thresholding these $\hat{\gamma_{i,j}}^{(\ell)}$ values

Simulation Results



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- Understanding of genetic basis of heritability can lead to better treatment

Thanks for listening

Questions? Comments?